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Action with Fractions is Contained in Division.

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This booklet, one of a series has been developed for the project, A Program for Mathematically Underdeveloped Pupils. A project team, including inservice teachers, is being used to write and develop the materials for this program. The materials developed in this booklet include the idea of reciprocals, and counting fractional parts. Several mathematical problems involving the application of the above principles are included in the sections designed for student activity. Accompanying these booklets will be a "Teaching Strategy Booklet" which will include a description of teacher techniques, methods, suggested sequences, academic games, and suggested visual materials. (RP)

ACTION WITH FRACTIONS

IS

CONTAINED

IN

DIVISION

ESEA Title III

PROJECT MATHEMATICS

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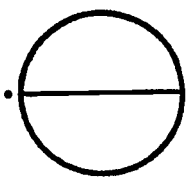
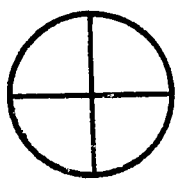
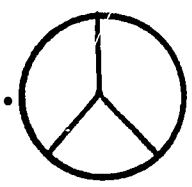
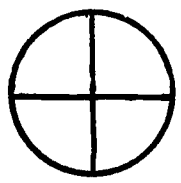
Reciprocals

In the unit on multiplication of fractions, the idea of reciprocals was examined. A more "dashing" name for reciprocals was given. Do you recall the other name? It was multiplicative inverses.

A pair of numbers is called "reciprocals" or "multiplicative inverses" if their product is one. First, review the idea by completing the following activity.

A model, given a count or measure of one, is drawn. A fraction is given, and the problem is to "count" how many times the fraction "is contained in" the one unit. Do this to supply the answer to the division problem. The first two are examples. (Check each division by multiplying.)

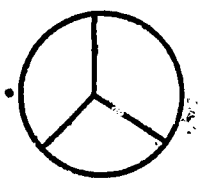
Model	Fraction	In One Unit	Division Problem
1. 	$\frac{1}{2}$	2 (times)	$1 \div \frac{1}{2} = 2$ Check $\frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1$
2. 	$\frac{3}{4}$	$1\frac{1}{3}$ or $\frac{4}{3}$	$1 \div \frac{3}{4} = \frac{4}{3}$ Check $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$
3. 	$\frac{1}{3}$	_____	$1 \div \frac{1}{3} = \square$ Check $\frac{1}{3} \times \square = 1$

4. 

$\frac{1}{4}$ _____ $1 \div \frac{1}{4} = \square$

Check

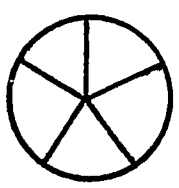
$\frac{1}{4} \times \square = 1$

5. 

$\frac{2}{3}$ _____ or _____ $1 \div \frac{2}{3} = \square$

Check

$\frac{2}{3} \times \square = 1$

6. 

$\frac{4}{5}$ _____ or _____ $1 \div \frac{4}{5} = \square$

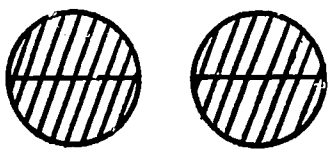
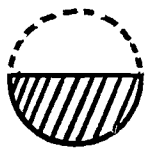
Check

$\frac{4}{5} \times \square = 1$

Counting Fractional Parts

For each activity below, a division model is drawn and the division problem is given beside the model. "Count" the number of times the fraction "is contained" in the model to obtain the answer to each division problem.

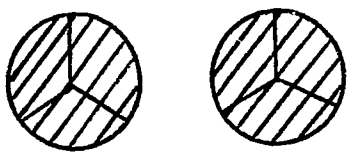
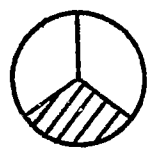
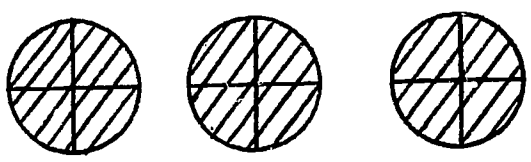
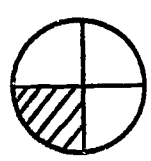
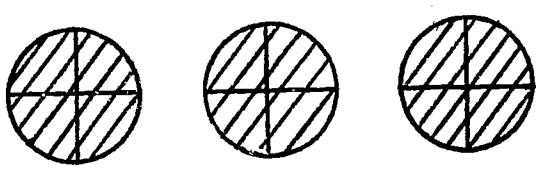
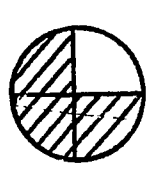
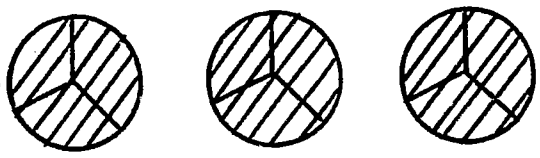
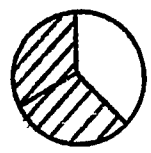
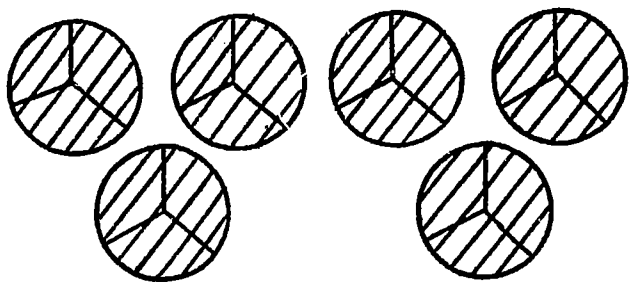
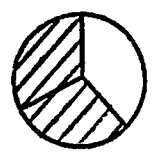
The first two are examples.

	Model	Fraction	Division
1.			$2 \div \frac{1}{2} = 4$

How many times is $\frac{1}{2}$ contained in 2?

Do you count 4 ?

Then $\frac{1}{2}$ is contained in 2 four times.

	Model	Fraction	Division
2.			$2 \div \frac{1}{3} = 6$
3.			$3 \div \frac{1}{4} =$
4.			$3 \div \frac{3}{4} =$
5.			$3 \div \frac{2}{3} =$
6.			$6 \div \frac{2}{3} =$

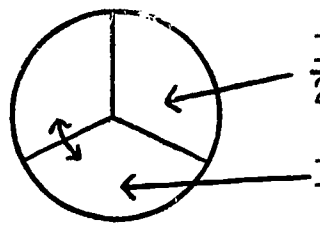
(How many $\frac{1}{4}$'s are there in 3?)

Now is the time to apply the idea of a reciprocal. Remember that a reciprocal of a number tells "how many times" the number is contained in one unit. (The product is one.) Consider the following problem:

First, count the number of times the fraction appears in one unit (its reciprocal.) Then multiply by the number of units.

Example: $3 \div \frac{2}{3} = \square$

First, count the number of $\frac{2}{3}$'s in one unit.



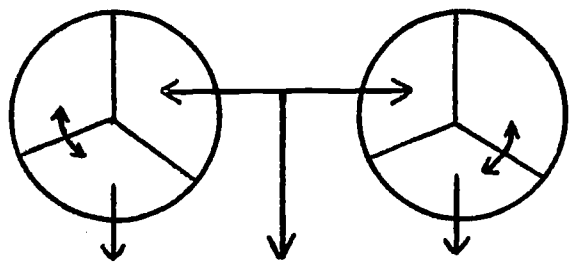
Answer: $1\frac{1}{2}$ or $\frac{3}{2}$

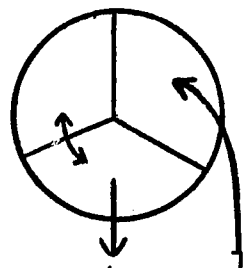
Then, in three units, the answer is: $3 \div \frac{2}{3} =$

$3 \times 1\frac{1}{2} =$

$3 \times \frac{3}{2} = \frac{9}{2} = \underline{\underline{4\frac{1}{2}}}$

Show this by counting $\frac{2}{3}$'s in three units.

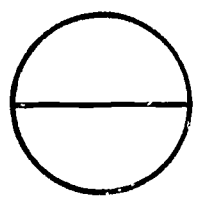




$= \underline{\underline{4\frac{1}{2}}}$

Try this for: $6 \div \frac{1}{2} =$ _____

First, the number of $\frac{1}{2}$'s in one unit?



Answer: 2

Then, in 6 units: $6 \times \frac{2}{1} = \underline{12}$ (One-half is in 6 units 12 times.)

Notice that in the division problems you simply multiply by the reciprocal of the divisor.

The reciprocal tells how many times the divisor is in one unit.

Example: $16 \div \frac{3}{4} =$

$16 \times \frac{4}{3} = \frac{64}{3}$

$= 21\frac{1}{3}$

(Three-fourths is in 16 units 21 and $\frac{1}{3}$ times.)

Activities

Complete each problem below. Show your answer is true for the first two problems by drawing a model and counting.

1. $6 \div \frac{3}{4} =$ ☐

2. $8 \div \frac{1}{2} =$ ☐

3. $3 \div \frac{5}{8} =$ ☐

4. $4 \div \frac{3}{5}$ ☐

5. $9 \div \frac{2}{3}$ ☐

6. $12 \div \frac{5}{6}$ ☐

7. $15 \div \frac{4}{5} =$ ☐

8. $10 \div \frac{1}{8} =$ ☐

9. $7 \div \frac{7}{12} =$ ☐

10. $5 \div \frac{8}{15}$ ☐

A second way of looking at dividing by a fraction is to use the idea of multiplying the divisor and the dividend by the same number (except zero). The quotient will remain unchanged.

Examine this idea below. Remember that division can be written as:

$$\frac{10}{2} = 10 \div 2$$

(We know the answer is 5.)

Now multiply both the numerator and the denominator by the same number.

$$\frac{10}{2} \times \frac{5}{5} = \frac{50}{10} = \square$$

$$\frac{10}{2} \times \frac{6}{6} = \frac{60}{12} = \square$$

$$\frac{10}{2} \times \frac{4}{4} = \frac{40}{8} = \square$$

Can you see why the quotient is unchanged?

Is it the same as multiplying by one?

Are $\frac{5}{5}$, $\frac{6}{6}$, $\frac{4}{4}$, etc. all names for one?

You are right. It is equivalent to multiplying by one.

Now let us rewrite the division problem:

$16 \div \frac{2}{3} = \square$ as $\frac{16}{\frac{2}{3}} = \square$

Since we know how to divide by a whole number, why not multiply the above divisor (two-thirds) so that the product is a whole number. Then multiply the numerator (16) by the same thing.

What can you multiply times two-thirds to get a whole number?

$$\_\_\_\_ \times \frac{2}{3} = \text{a whole number}$$

Did you pick any of these answers?

{3, 6, 9, 12, 15, ...}

Try some of these:

$$\frac{16}{\frac{2}{3}} = \underline{\underline{24}}$$

$$\frac{16}{\frac{2}{3}} \times \frac{(3)}{(3)} = \frac{\square}{\square} = \square$$

$$\frac{16}{\frac{2}{3}} \times \frac{(6)}{(6)} = \frac{\square}{\square} = \square$$

$$\frac{16}{\frac{2}{3}} \times \frac{(9)}{(9)} = \frac{\square}{\square} = \square$$

All the above can be rewritten as:

$$16 \div \frac{2}{3} = \square$$

$$(16 \times 3) \div (\tfrac{2}{3} \times 3) =$$

$$48 \div 2 = \square$$

Actually, the simplest divisor to use would be <u>one</u>.

What can you multiply the divisor by to get a product of <u>one</u>? You are right if you say its reciprocal. Then:

$$\frac{16}{\frac{2}{3}} \text{ is equal to: } \frac{16}{\frac{2}{3}} \times \frac{\frac{3}{2}}{\frac{3}{2}} = \frac{\frac{48}{2}}{1} = 24$$

OR:

$$16 \div \frac{2}{3} \text{ is equal to: } (16 \times \tfrac{3}{2}) \div (\tfrac{2}{3} \times \tfrac{3}{2}) = \frac{48}{2} \div 1 = 24$$

Actually all this is not necessary if you want the fastest way to write down the divisor times its reciprocal, since the answer will always be one.

Then: $16 \div \frac{2}{3} =$

$16 \times \frac{3}{2} =$

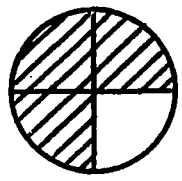
$\frac{48}{2} = \underline{24}$

<u>This is the fastest way</u>.

Try a fraction divided by a fraction:

$\frac{3}{4} \div \frac{1}{2} = \square$ (How many one-halves are in three-fourths?)

First count:



By counting do you see <u>one</u> one-half and $\frac{1}{2}$ of another one-half?

Then: $\frac{3}{4} \div \frac{1}{2} = \underline{1\ \frac{1}{2}}$

<u>By the faster method:</u>

$\frac{3}{4} \div \frac{1}{2} =$

$\frac{3}{4} \times \frac{2}{1} =$

$\frac{6}{4} = 1\ \frac{2}{4} = \underline{1\ \frac{1}{2}}$

Activities

Complete as many problems as you can without paper or pencil.

1. $\frac{3}{4} \div \frac{1}{2} = \square$
2. $\frac{5}{8} \div \frac{2}{3} = \square$
3. $\frac{1}{3} \div \frac{6}{7} = \square$
4. $\frac{1}{4} \div \frac{3}{5} = \square$
5. $\frac{5}{6} \div \frac{7}{8} = \square$
6. $\frac{5}{7} \div \frac{1}{2} = \square$
7. $\frac{3}{8} \div \frac{3}{4} = \square$
8. $\frac{3}{7} \div \frac{2}{3} = \square$
9. $\frac{1}{6} \div \frac{1}{4} = \square$
10. $\frac{1}{7} \div \frac{7}{8} = \square$

The next activity involves dividing a fraction by a whole number. The ideas used in solving other problems will work. For example:

$$\frac{1}{2} \div \frac{3}{1}$$

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

One way to picture this problem is to take $\frac{1}{2}$ and cut it into 3 equal parts. One of these three parts is the answer.

Notice the answer is $\frac{1}{6}$ of the whole or just plain $\frac{1}{6}$.

Now look at the two problems below:

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

and

$$\frac{1}{2} \div \frac{3}{1} = \frac{1}{6}$$

Examine these problems:

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{4} \div 2$

$\frac{1}{6} \times \frac{1}{3} = \frac{1}{6} \div 3$

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{2} \div 4$

Do you notice the relation between multiplication of fractions and division? They are <u>inverse</u> operations.

Activities

See how many you can complete mentally.

1. $\frac{3}{4} \div 2 =$ ☐
2. $\frac{5}{8} \div 5 =$ ☐
3. $\frac{1}{2} \div 12 =$ ☐
4. $\frac{3}{5} \div 8 =$ ☐
5. $\frac{5}{6} \div 20 =$ ☐
6. $\frac{2}{3} \div 7 =$ ☐
7. $\frac{1}{4} \div 3 =$ ☐
8. $\frac{2}{5} \div 15 =$ ☐
9. $\frac{1}{6} \div 17 =$ ☐
10. $\frac{3}{7} \div 6 =$ ☐

In division, where mixed numbers are involved, it is easier to "rename" and then divide. For example:

I: $6 \div 2\frac{1}{3} =$

$6 \div \frac{7}{3} =$

$6 \times \frac{3}{7} = \frac{18}{7} = 2\frac{4}{7}$

II: $3\frac{1}{3} \div 2\frac{1}{2} =$

$\frac{10}{3} \times \frac{2}{5} =$

$\frac{20}{15} = 1\frac{5}{15} = 1\frac{1}{3}$

Activities

Work the following problems. You might want to look again at examples I and II on the preceding page.

1. $2\frac{1}{2} \div 1\frac{3}{4} =$ _____

2. $3\frac{2}{3} \div 5\frac{1}{5} =$ _____

3. $4\frac{5}{6} \div 7\frac{1}{8} =$ _____

4. $6\frac{1}{6} \div 8\frac{1}{4} =$ _____

5. $9\frac{1}{2} \div 10\frac{1}{5} =$ _____

6. $15\frac{1}{3} \div 12\frac{2}{3} =$ _____

7. $18\frac{1}{2} \div 9\frac{2}{5} =$ _____

8. $11\frac{2}{3} \div 2\frac{2}{5} =$ _____

9. $12\frac{3}{5} \div 20\frac{1}{2} =$ _____

10. $15\frac{3}{8} \div 6\frac{3}{4} =$ _____

Activities

This exercise includes all the different kinds of fraction division problems you've worked on in this unit. Solve some as a class activity and complete the others as individual work.

1. $7 \div \frac{5}{8} =$ _____

2. $\frac{3}{4} \div \frac{7}{8} =$ _____

3. $1\frac{1}{2} \div \frac{3}{4} =$ _____

4. $12 \div \frac{3}{4} =$ _____

5. $\frac{1}{2} \div 15 =$ _____

6. $\frac{5}{6} \div \frac{9}{10} =$ _____

7. $5\frac{7}{8} \div \frac{1}{2} =$ _____

8. $9 \div \frac{1}{2} =$ _____

9. $\frac{1}{7} \div \frac{11}{12} =$ _____

10. $\frac{6}{7} \div 12 =$ _____

11. $7\frac{3}{4} \div \frac{5}{6} =$ _____

12. $3 \div \frac{7}{8} =$ _____

13. $\frac{3}{4} \div 10 =$ _____

14. $\frac{2}{3} \div \frac{11}{15} =$ _____

15. $8 \div \frac{2}{3} =$

16. $4\frac{2}{3} \div \frac{3}{4} =$ _____

17. $\frac{6}{7} \div \frac{1}{2} =$ _____

18. $15 \div \frac{5}{8} =$ _____

19. $\frac{2}{3} \div 4 =$ _____

20. $\frac{3}{8} \div \frac{8}{11} =$ _____

21. $1\frac{14}{15} \div \frac{2}{3} =$ _____

22. $4 \div \frac{5}{12} =$ _____

23. $\frac{7}{12} \div 13 =$ _____

24. $\frac{3}{4} \div \frac{2}{3} =$ _____

25. $5\frac{2}{3} \div \frac{7}{8} =$ _____

26. $10 \div \frac{5}{9} =$ _____

27. $\frac{1}{6} \div \frac{1}{2} =$ _____

28. $\frac{4}{5} \div 15 =$ _____

29. $10\frac{7}{8} \div \frac{2}{9} =$ _____

30. $\frac{1}{2} \div \frac{14}{15} =$ _____